Name ___Key___

Show All Work
NO GRAPHING CALCULATORS

1) Sketch the graph of $f(x) = x(x-1)^2(x+1)^2$. Make sure your graph shows all intercepts and exhibits the proper end behavior.
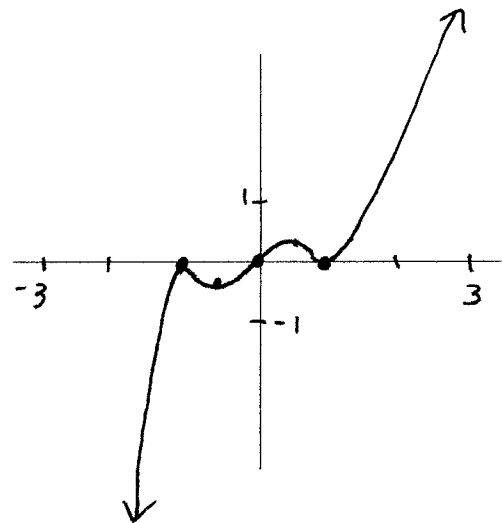
$f(x)$ behaves like $y = x^5$

x intercepts $0, 1, -1$

$f(-2) = (-2)(9)(1) = -18$

$f(-\frac{1}{2}) = -\frac{1}{2}(\frac{9}{4}) \cdot \frac{1}{4} = \frac{-9}{32}$

$f(\frac{1}{2}) = \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{9}{4} = \frac{9}{32}$

$f(2) = 2 \cdot 1 \cdot 9 = 18$

2) Find the quotient and remainder using long division. $\dfrac{x^3+6x+3}{x^2-2x+2}$

$$
\begin{array}{r}
x + 2 \\
x^2-2x+2 \overline{\smash{\big)} x^3 \qquad +6x+3} \\
\underline{x^3-2x^2+2x} \\
2x^2+4x+3 \\
\underline{2x^2-4x+4} \\
8x-1
\end{array}
$$

$Q(x) = x+2$

$R(x) = 8x-1$

3) Find the vertical and horizontal asymptotes of the following function.

$R(x) = \dfrac{(x+1)(x-2)}{(x+5)(x+1)}$

Vert. Asy $x = -5$ (denominator $= 0$
                          even when simplified)

Horiz Asy $y = 1$ (degrees numerator and
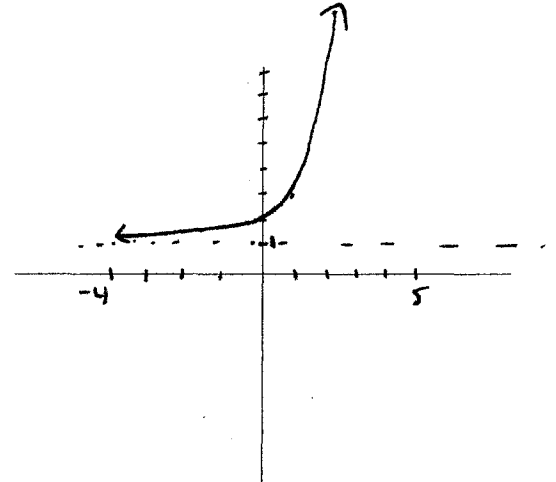   denominator are the same)

4) If f(x) = 2^x find

a) f(0) $= 2^0 = 1$

b) f(-1) $= 2^{-1} = \frac{1}{2}$

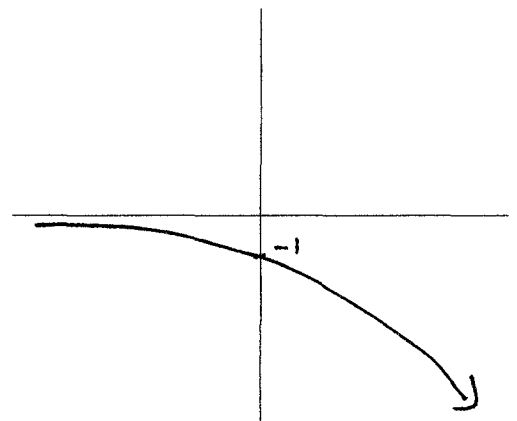c) f(1/2) $= 2^{1/2} = \sqrt{2} \doteq 1.41$

5) Graph $f(x) = 2^x + 1$

6) If $10,000 is invested at 3% per year compounded quarterly, how much will be in the bank after 5 years. Recall $A(t) = P(1 + \frac{r}{n})^{nt}$

$$A(5) = 10,000 \left(1 + \frac{.03}{4}\right)^{20} = \$ 11,611.84$$

7) If f(x) = e^x what is f(2)?

$$f(2) \doteq 7.389056\ldots$$

8) Sketch the graph of f(x) = - e^x

9) Doctors use radioactive iodine as a tracer in diagnosing certain thyroid gland disorders. This type of iodine decays in such a way that the mass remaining after t days is given by the function $m(t) = 6e^{-0.087t}$ where m(t) is measured in grams.

a) Find the mass at time t = 0

$$m(0) = 6e^{0} = 6$$

b) How much of the mass remains in 20 days?

$$m(20) = 6e^{(-0.087)(20)} = 6e^{-1.74} \approx 1.053$$

10) a) Put $\log_5 25 = 2$ in exponential form

$$5^2 = 25$$

b) Put $3^2 = 9$ in logarithmic form

$$\log_3 9 = 2$$

c) Put $x^y = z$ in logarithmic form

$$\log_x z = y$$

11) Use the definition of the logarithmic function to find x.
a) $\log_2 16 = x$

$$2^x = 16 \rightarrow \boxed{x = 4}$$

b) $\log_x 64 = 3$ $\rightarrow$ $x^3 = 64 \rightarrow \boxed{x = 4}$

c) $\log_2 x = 5$ $\rightarrow$ $2^5 = x \rightarrow \boxed{x = 32}$

12) What is the domain of $f(x) = \ln(3 - x)$?

$$3 - x > 0$$
$$3 > x$$
$$x < 3$$

$$D = \{x \mid x < 3\}$$
$$= (-\infty, 3)$$

13) Sketch the graph of $f(x) = \ln x$